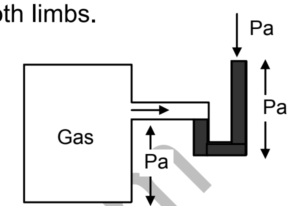
SOLUTIONS TO CONCEPTS
CHAPTER 13

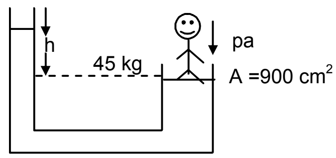
1. $p = h \rho g$
   It is necessary to specify that the tap is closed. Otherwise pressure will gradually decrease, as h decrease, because, of the tap is open, the pressure at the tap is atmospheric.

2. a) Pressure at the bottom of the tube should be same when considered for both limbs.
   From the figure are shown,
   $p_g + \rho_{Hg} \times h_2 \times g = p_a + \rho_{Hg} \times h_1 \times g$
   $\Rightarrow \quad p_g = p_a + \rho_{Hg} \times g(h_1 - h_2)$



   b) Pressure of mercury at the bottom of u tube
   $p \ = p_a + \rho_{Hg}\, h_1 \times g$

3. From the figure shown
   $p_a + h\rho g = p_a + mg/A$
   $\Rightarrow h\rho g = mg/A$
   $\Rightarrow h = \dfrac{m}{A\rho}$



4. a) Force exerted at the bottom.
   = Force due to cylindrical water colum + atm. Force
   $= A \times h \times \rho_w \times g + p_a \times A$
   $= A(h\, \rho_w\, g + p_a)$
   b) To find out the resultant force exerted by the sides of the glass, from the freebody, diagram of water inside the glass
   $p_a \times A + mg = A \times h \times \rho_w \times g + F_s + p_a \times A$
   $\Rightarrow mg = A \times h \times \rho_w \times g + F_s$
   This force is provided by the sides of the glass.

5. If the glass will be covered by a jar and the air is pumped out, the atmospheric pressure has no effect.
   So,
   a) Force exerted on the bottom.
   $= (h\, \rho_w\, g) \times A$
   b) $mg = h \times \rho_w \times g \times A \times F_s$.
   c) It glass of different shape is used provided the volume, height and area remain same, no change in answer will occur.

6. Standard atmospheric pressure is always pressure exerted by 76 cm Hg column
   $= (76 \times 13.6 \times g)$ Dyne/cm^2.
   If water is used in the barometer.
   Let h $\rightarrow$ height of water column.
   $\therefore h \times \rho_w \times g$

7. a) $F = P \times A = (h\, \rho_w \times g)\, A$
   b) The force does not depend on the orientation of the rock as long as the surface area remains same.

8. a) $F = A\, h\, \rho\, g$.
   b) The force exerted by water on the strip of width δx as shown,
   $dF = p \times A$
   $= (x\rho g) \times A$
   c) Inside the liquid force act in every direction due to adhesion.
   $di = F \times r$
   d) The total force by the water on that side is given by
   $$F = \int_0^1 20000\, x\delta x \Rightarrow F = 20{,}000\ [x^2/2]_0^1$$
   e) The torque by the water on that side will be,

$$i = \int_0^1 20000\, x\delta x\,(1 - x) \Rightarrow 20{,}000\ [x^2/2 - x^3/3]_0^1$$

9. Here, $m_0 = m_{Au} + m_{cu} = 36$ g …(1)
   Let V be the volume of the ornament in cm^3
   So, $V \times \rho_w \times g = 2 \times g$
   $\Rightarrow (V_{au} + V_{cu}) \times \rho_w \times g = 2 \times g$
   $\Rightarrow \left(\frac{m}{\rho_{au}} + \frac{m}{\rho_{au}}\right)\rho_w \times g = 2 \times g$
   $\Rightarrow \left(\frac{m_{Au}}{19.3} + \frac{m_{Au}}{8.9}\right) \times 1 = 2$
   $\Rightarrow 8.9\, m_{Au} + 19.3\, m_{cu} = 2 \times 19.3 \times 8.9 = 343.54$ …(2)
   From equation (1) and (2), $8.9\, m_{Au} + 19.3\, m_{cu} = 343.54$
   $\Rightarrow \frac{8.9(m_{Au} + m_{cu}) = 8.9 \times 36}{m_{cu} = 2.225\text{g}}$
   So, the amount of copper in the ornament is 2.2 g.
10. $\left(\frac{M_{Au}}{\rho_{Au}} + V_c\right)\rho_w \times g = 2 \times g$ (where V_c = volume of cavity)
11. mg = U + R (where U = Upward thrust)
   $\Rightarrow$ mg – U = R
   $\Rightarrow R = mg - v\,\rho_w\, g$ (because, $U = v\rho_w g$)
   $= mg - \frac{m}{\rho} \times \rho_w \times g$
12. a) Let $V_i \rightarrow$ volume of boat inside water = volume of water displace in m^3.
   Since, weight of the boat is balanced by the buoyant force.
   $\Rightarrow mg = V_i \times \rho_w \times g$
   b) Let, $v^1 \rightarrow$ volume of boat filled with water before water starts coming in from the sides.
   $mg + v^1\,\rho_w \times g = V \times \rho_w \times g$.
13. Let $x \rightarrow$ minimum edge of the ice block in cm.
   So, $mg + W_{ice} = U$. (where U = Upward thrust)
   $\Rightarrow 0.5 \times g + x^3 \times \rho_{ice} \times g = x^3 \times \rho_w \times g$
14. $V_{ice} = V_k + V_w$
   $V_{ice} \times \rho_{ice} \times g = V_k \times \rho_k \times g + V_w \times \rho_w \times g$
   $\Rightarrow (V_k + V_w) \times \rho_{ice} = V_k \times \rho_k + V_w \times \rho_w$
   $\Rightarrow \frac{V_w}{V_k} = 1$.
15. $V_{ii}g = V\,\rho_w\, g$
16. $(m_w + m_{pb})g = (V_w + V_{pb})\,\rho \times g$
   $\Rightarrow (m_w + m_{pb}) = \left(\frac{m_w}{\rho_w} + \frac{m_{pb}}{\rho_{pb}}\right)\rho$
17. $Mg = w \Rightarrow (m_w + m_{pb})g = V_w \times \rho \times g$
18. Given, x = 12 cm
   Length of the edge of the block $\rho_{Hg} = 13.6$ gm/cc
   Given that, initially 1/5 of block is inside mercuty.
   Let $\rho_b \rightarrow$ density of block in gm/cc.
   $\therefore (x)^3 \times \rho_b \times g = (x)^2 \times (x/5) \times \rho_{Hg} \times g$
   $\Rightarrow 12^3 \times \rho_b = 12^2 \times 12/5 \times 13.6$
   $\Rightarrow \rho_b = \frac{13.6}{5}$ gm/cc

After water poured, let x = height of water column.

$V_b = V_{Hg} + V_w = 12^3$

Where V_{Hg} and V_w are volume of block inside mercury and water respectively

$\therefore (V_b \times \rho_b \times g) = (V_{Hg} \times \rho_{Hg} \times g) + (V_w \times \rho_w \times g)$

$\Rightarrow (V_{Hg} + V_w)\rho_b = V_{Hg} \times \rho_{Hg} + V_w \times \rho_w.$

$\Rightarrow (V_{Hg} + V_w) \times \dfrac{13.6}{5} = V_{Hg} \times 13.6 + V_w \times 1$

$\Rightarrow (12)^3 \times \dfrac{13.6}{5} = (12 - x) \times (12)^2 \times 13.6 + (x) \times (12)^2 \times 1$

$\Rightarrow x = 10.4$ cm

19. Here, Mg = Upward thrust

$\Rightarrow V\rho g = (V/2)(\rho_w) \times g$ (where ρ_w = density of water)

$\Rightarrow \left(\dfrac{4}{3}\pi r_2^3 - \dfrac{4}{3}\pi r_1^3\right)\rho = \left(\dfrac{1}{2}\right)\left(\dfrac{4}{3}\pi r_2^3\right) \times \rho_w$

$\Rightarrow (r_2^3 - r_1^3) \times \rho = \dfrac{1}{2} r_2^3 \times 1 = 865$ kg/m^3.

20. $W_1 + W_2 = U.$

$\Rightarrow mg + V \times \rho_s \times g = V \times \rho_w \times g$ (where ρ_s = density of sphere in gm/cc)

$\Rightarrow 1 - \rho_s = 0.19$

$\Rightarrow \rho_s = 1 - (0.19) = 0.8$ gm/cc

So, specific gravity of the material is 0.8.

21. $W_i = mg - V_i\, \rho_{air} \times g = \left(m - \dfrac{m}{\rho_i}\rho_{air}\right)g$

$W_w = mg - V_w\, \rho_{air}\, g = \left(m - \dfrac{m}{\rho_w}\rho_{air}\right)g$

22. Driving force $U = V\rho_w g$

$\Rightarrow a = \pi r^2 (X) \times \rho_w\, g \;\Rightarrow T = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{Acceleration}}}$

23. a) $F + U = mg$ (where $F = kx$)

$\Rightarrow kx + V\rho_w g = mg$

b) $F = kX + V\rho_w \times g$

$\Rightarrow ma = kX + \pi r^2 \times (X) \times \rho_w \times g = (k + \pi r^2 \times \rho_w \times g)X$

$\Rightarrow \omega^2 \times (X) = \dfrac{(k + \pi r^2 \times \rho_w \times g)}{m} \times (X)$

$\Rightarrow T = 2\pi\sqrt{\dfrac{m}{K + \pi r^2 \times \rho_w \times g}}$

24. a) $mg = kX + V\rho_w g$

b) $a = kx/m$

$w^2 x = kx/m$

$T = 2\pi\sqrt{m/k}$

25. Let $x \rightarrow$ edge of ice block

When it just leaves contact with the bottom of the glass.

$h \rightarrow$ height of water melted from ice

$W = U$

$\Rightarrow x^3 \times \rho_{ice} \times g = x^2 \times h \times \rho_w \times g$

Again, volume of water formed, from melting of ice is given by,

$4^3 - x^3 = \pi \times r^2 \times h - x^2 h$ (because amount of water $= (\pi r^2 - x^2)h$)

$\Rightarrow 4^3 - x^3 = \pi \times 3^2 \times h - x^2 h$

Putting $h = 0.9\,x \Rightarrow x = 2.26$ cm.

26. If $p_a \rightarrow$ atm. Pressure
    $A \rightarrow$ area of cross section
    $h \rightarrow$ increase in hright
    $p_aA + A \times L \times \rho \times a_0 = pa^A + h\rho g \times A$
    $\Rightarrow hg = a_0L \quad \Rightarrow \quad a_0L/g$
27. Volume of water, discharged from Alkananda + vol are of water discharged from Bhagirathi = Volume of water flow in Ganga.
28. a) $a_A \times V_A = Q_A$
    b) $a_A \times V_A = a_B \times V_B$
    c) $1/2\ \rho v_A^2 + p_A = 1/2\ \rho v_B^2 + p_B$
    $\Rightarrow (p_A - p_B) = 1/2\ \rho\ (v_B^2 - v_A^2)$
29. From Bernoulli's equation, $1/2\ \rho v_A^2 + \rho g h_A + p_A$
    $= 1/2\ \rho v_B^2 + \rho g h_B + p_B$.
    $\Rightarrow P_A - P_B = (1/2)\ \rho\ (v_B^2 - v_A^2) + \rho g\ (h_B - h_A)$
30. $1/2\ \rho v_B^2 + \rho g h_B + p_B = 1/2\ \rho v_A^2 + \rho g h_A + p_A$
31. $1/2\ \rho v_A^2 + \rho g h_A + p_A = 1/2\ \rho v_B^2 + \rho g h_B + p_B$
    $\Rightarrow P_B - P_A = 1/2\ \rho(v_A^2 - v_B^2) + \rho g\ (h_A - h_B)$
32. $\vec{v}_A a_A = \vec{v}_B \times a_B$
    $\Rightarrow 1/2\ \rho v_A^2 + \rho g h_A + p_A = 1/2\ \rho v_B^2 + \rho g h_B + p_B$
    $\Rightarrow 1/2\ \rho v_A^2 + p_A = 1/2\ \rho v_B^2 + p_B$
    $\Rightarrow P_A - P_B = 1/2\ \rho(v_B^2 - v_B^2)$
    Rate of flow $= v_a \times a_A$
33. $V_A\ a_A = v_B\ a_B \Rightarrow \frac{v_A}{B} = \frac{a_B}{a_A}$

    $5v_A = 2v_B \Rightarrow v_B = (5/2)v_A$
    $1/2\ \rho v_A^2 + \rho g h_A + p_A = 1/2\ \rho v_B^2 + \rho g h_B + p_B$
    $\Rightarrow P_A - P_B = 1/2\ \rho\ (v_B^2 - v_B^2)$ (because $P_A - P_B = h\rho_m g$)
34. $P_A + (1/2)\rho v_A^2 = P_B + (1/2)\ \rho v_B^2 \Rightarrow p_A - p_B = (1/2)\rho v_B^2\ \{v_A = 0\}$
    $\Rightarrow \rho g h = (1/2)\ \rho v_B^2\ \{p_A = p_{atm} + \rho g h\}$
    $\Rightarrow v_B = \sqrt{2gh}$
    a) $v = \sqrt{2gh}$
    b) $v = \sqrt{2g(h/2)} = \sqrt{gh}$
    c) $v = \sqrt{2gh}$
    $v = av \times dt$
    $AV = av$
    $\Rightarrow A \times \frac{dh}{dt} = a \times \sqrt{2gh} \Rightarrow dh = \frac{a \times \sqrt{2gh} \times dt}{A}$
    d) $dh = \frac{a \times \sqrt{2gh} \times dt}{A} \Rightarrow T = \frac{A}{a}\sqrt{\frac{2}{g}}[\sqrt{H_1} - \sqrt{H_2}]$
35. $v = \sqrt{2g(H-h)}$
    $t = \sqrt{2h/g}$
    $x = v \times t = \sqrt{2g(H-h) \times 2h/g} = 4\sqrt{(Hh - h^2)}$
    So, $\Rightarrow \left(\frac{d}{dh}\right)(Hh - h^2) = 0 \Rightarrow 0 = H - 2h \Rightarrow h = H/2$.

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